Annotated Bibliography for
The Physicalization of Metamathematics and Its Implications for the Foundations of Mathematics
Stephen Wolfram

The main text includes direct links to specific documents and references. Here we'll give a slightly more general bibliography, though most of it should be considered background, since the approach taken here represents a significant departure from traditional directions, and builds more or less directly on extremely low-level concepts.

The earliest known large-scale axiomatic presentation of mathematics was:
  Euclid (300 BC), Στοιχεῖα (in Ancient Greek) [Elements].

Empirical metamathematics from this was given in:

The concept that there is underlying reality in mathematics was discussed in:
  Plato (375 BC), πολιτεία (in Ancient Greek) [The Republic].
  Plato (360 BC), Τύμαιος (in Ancient Greek) [Timaeus].

Modern explorations of these ideas include:
  M. Balaguer (1998), Platonism and Anti-Platonism in Mathematics, Oxford University Press.

The contemporary axiomatic formulation of mathematics was developed in:
R. Dedekind (1888), *Was sind und was sollen die Zahlen?* (in German), F. Vieweg und Sohn. (Translated in H. Pogorzelski, et al. (1995), as *What Are Numbers and What Should They Be?*, Research Institute for Mathematics.)


D. Hilbert (1903), *Grundlagen der geometrie* (in German), B. G. Teubner. (Translated by E. G. Townsend (1902), as *The Foundations of Geometry*, Open Court.)


A standard collection of source documents is:


Among general commentaries on axiomatic formalism are:


W. Sieg (2013), Hilbert’s Programs and Beyond, Oxford University Press.


Expositions of metamathematics and mathematical logic ideas appear for example in:

D. Hilbert and W. Ackermann (1928), Grundzüge der theoretischen Logik (in German), Springer. (Translated by L. M. Hammond, et al. (2000), as Principles of Mathematical Logic, AMS Chelsea.)


H. Rasiowa (1963), The Mathematics of Metamathematics, Panstwowe Wydawnictwo Naukowe.


Works on the metamodeling of mathematics and on universal algebra include:

A. N. Whitehead (1898), *A Treatise on Universal Algebra with Applications*, Cambridge University Press.


Low-level symbolic representations of mathematics were developed in:


See also these commentaries:


Practical representation of mathematics using symbolic transformations was developed in:


A proof of the “arithmeticization of metamathematics” was given in:


A major precursor to the current work is:


Our Physics Project is described in:

S. Wolfram (2020), A Project to Find the Fundamental Theory of Physics, Wolfram Media.


The concept of the ruliad was introduced in:


The concept of the mathematical observer was introduced in:


Automated theorem proving is discussed for example in:


Relevant Wolfram Language functions include:

A system for low-level proof-oriented formalized mathematics (used here for empirical metamathematics) is:


Other systems for proof-oriented formalized mathematics include:

N. G. de Bruijn (1967), Automath [Software system]. win.tue.nl/automath.
A. Trybulec (1973), Mizar [Software system]. mizar.uwb.edu.pl.
University of Cambridge and Technical University of Munich (1986), Isabelle [Software system]. isabelle.in.tum.de.
Microsoft Research (2013), Lean [Software system]. leanprover.github.io.

Discussions of formalized mathematics include:


Books on the philosophy of mathematics and its foundations include:


The philosophical study of mathematics in terms of interrelated structure is summarized in:


The structure of proof space is discussed in univalent foundations and homotopy type theory:

The Univalent Foundations Program (2013), *Homotopy Type Theory*, Institute for Advanced Study.


Relations between category theory, mathematics and our Physics Project were explored in:
