STANFORD UNIVERSITY
STANFORD, CALIFORNIA 94305

DEPARTMENT OF MATHEMATICS

October 1971.

Theorem.

Let $X=S(SI(S(SI(KS))(K(KK))))(KK)=\lambda x.x(xS(KK))K$.

Then $XXX=K$ and $XK=S$. Hence $\{X\}$ is a basis for the combinators.

Proof.

Define $K_1=K$, $K_{n+1}=KK_n$. Then $K_{n+1}x=K_n$.

We calculate:

$XSK_2=S(SSK_2)KK_2=SSK_2K_2(KK_2)=SK_2(K_2K_2)(K_3)=K_2K_3(K_1K_3)=K_1K_4=K_5$.

$XX=X(XS(KK))K=X(XSK_2)K=XK_5K=K_5(K_5S(KK))KK=K_4KK=K_3K=K_2$.

$XXX=K_2X=K$.

$XK=K(KSK_2)K=KSK_2=S$.

The following simplification is due to C. Böhm:

Define $X=\lambda x.x(xSA)K$, where $A=K^3I$.

Then $XX=K$ and $XK=S$.

Henk Barendregt.

THE UNIVERSITY OF WISCONSIN
MADISON 53706

MATHEMATICS RESEARCH CENTER
J. BARKLEY ROSSER, DIRECTOR

TELEPHONE
(608) 262-3636

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Dr. Henk Barendregt
Department of Mathematics
Stanford University
Stanford, California 94305

Dear Dr. Barendregt,

I have your note of October 1971, with your example of an X such that

XXX --> K,
XK --> S,

and Bohm's example of an X such that

XX --> K
XK --> S.

Following Bohm, one can take

$A = K^4I = K(K(K(KI)))$,

$X = \lambda x.x\,ASK = S(S(SI(KA))(KS))(KK)$.

Then

$XX \to XASK \to AASKSK \to K^3ISKSK \to K^2IKSK \to KISK \to IK \to K$,

$X(KK) \to KKASK \to KSK \to S$.

Alternatively, one can take

$X = \lambda x.x\,KSK = S(S(SI(KK))(KS))(KK)$.

Then

XX --> XKSK --> KKSKSK --> KKSK --> KK,

XXX --> KKX --> K,

X(XX) --> X(KK) --> KKKSK --> KSK --> S.

You are no doubt aware that Schönfinkel proposed to define all the combinators in terms of a J with the property

JJ = S , JS = K.

See pp. 356 and 366 of the enclosed copy. It is very easy to construct a J with the properties indicated above. Take A to be K(KK). Then, whatever X and Y are,

AXY --> KKY --> K.

Then take

J = λx . x AK(KS) = S(S(SI(KA))(KK))(K(KS)). <k^2k, k, ks>

We get

JJ --> JAK(KS) --> AAK(KS)K(KS) --> K(KS)K(KS) --> KS(KS) --> S,

JS --> SAK(KS) --> A(KS)(K(KS)) --> K.

Sincerely,

J. Barkley Rosser

J. Barkley Rosser
Director

Encl.
cc: Professor H. B. Curry
Dean S. C. Kleene

Let $X \equiv \langle K^*, J, I \rangle$ where K^* is a local K for $\{I, J, X\}$. Take e.g.

$K^* \equiv \lambda xy.\, y IIII x$.

Then $\{X\}$ is a basis for the λI-calculus:

$XX \to XK^*JI \to K^*K^*JII \to$
$\to K^*IJI \to JII \to I$
$XI \to IK^*JI \to K^*JI \to J$.