Exercises

Lecture 1-TBD, Chapter 1

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Out[4]= {2026, 8, 28, 5, 35, 21.892808}

- *This notebook introduces you to **some exercises from Chapter 1***
- *With respect to any formulae, try **not to** memorize more than the most basic equations and notions. Anything more complex can always be developed from the fundamental relations, as needed.*

- *You can download and interact with the corresponding notebook within your copy of Mathematica on your computer or Mathematica Online-Basic with a free account.*

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1. Overview

This notebook attempts exercises from the textbook.

2. Exercises

2.1 Exercise 1.1-4 (pp 16)

2.1.1 Statement

1.1-4. Use the nodal equation method to find v_{out}/v_{in} of Fig. P1.1-4.

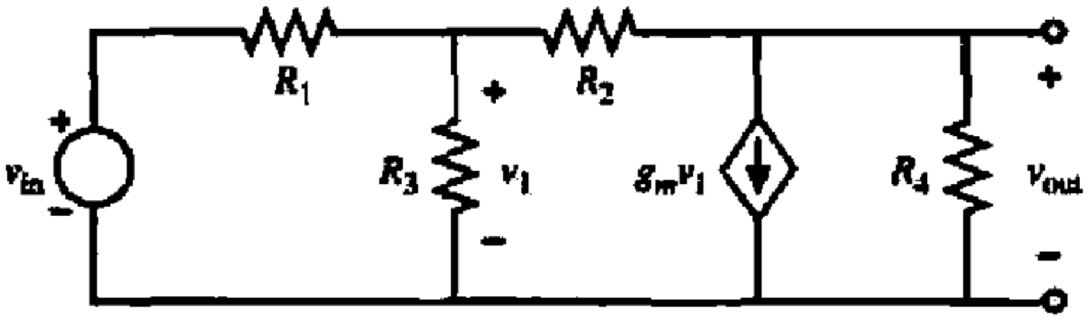


Figure P1.1-4

2.1.2 Solution

$$IR2 = gm * v_1 + \frac{v_{out}}{R4}$$

$$v_1 = v_{out} + IR2 * R2$$

$$v_1 = v_{out} + \left(gm * v_1 + \frac{v_{out}}{R4}\right) * R2$$

$$v_1 = v_{out} + gm * v_1 * R2 + \left(\frac{R2}{R4}\right) v_{out}$$

$$v_1 - gm * v_1 * R2 = v_{out} + \left(\frac{R2}{R4}\right) v_{out}$$

$$v_1\,(1 - gm * R2) = v_{out} + \left(\frac{R2}{R4}\right) v_{out} \tag{1}$$

Now express v_1 in terms of v_{in}.

$$v_1 = v_{in} - \left(IR2 + \frac{v_1}{R3}\right) R1$$

$$v_1 = v_{in} - \left(gm * v_1 + \frac{v_{out}}{R4} + \frac{v_1}{R3}\right) R1$$

$$v_1 = v_{in} - gm * v_1\, R1 - \frac{R1}{R4} v_{out} - \frac{R1}{R3} v_1$$

$$v_1 + gm * v_1 * R1 + \frac{R1}{R3} v_1 = v_{in} - \frac{R1}{R4} v_{out}$$

$$v_1 \left(1 + gm * R1 + \frac{R1}{R3}\right) = v_{in} - \frac{R1}{R4} v_{out}$$

$$v_1 = \frac{v_{in} - \frac{R1}{R4} v_{out}}{\left(1 + gm * R1 + \frac{R1}{R3}\right)} \tag{2}$$

Substitute for v_1 from (2) back into (1)

$$\frac{v_{in} - \frac{R1}{R4} v_{out}}{\left(1 + gm * R1 + \frac{R1}{R3}\right)} (1 - gm * R2) = v_{out} + \left(\frac{R2}{R4}\right) v_{out}$$

Let $A = \frac{(1-gm*R2)}{\left(1+gm*R1+\frac{R1}{R3}\right)}$.

$$A\, v_{in} - A \frac{R1}{R4} v_{out} = v_{out} + \left(\frac{R2}{R4}\right) v_{out}$$

Move terms around algebraically.

$$A\, v_{in} = v_{out} + \left(\frac{R2}{R4}\right) v_{out} + A \left(\frac{R1}{R4}\right) v_{out}$$

$$A\, v_{in} = v_{out} \left(1 + \left(\frac{R2}{R4}\right) + A \left(\frac{R1}{R4}\right)\right)$$

$$\frac{v_{out}}{v_{in}} = \frac{A}{\left(1 + \left(\frac{R2}{R4}\right) + A \left(\frac{R1}{R4}\right)\right)}$$

In[5]:=
```
myGain = Simplify[A/(1 + R2/R4 + A (R1/R4)) //. A -> (1 - gm * R2)/(1 + gm * R1 + R1/R3)]
```

Out[5]=
$$-\frac{(-1 + gm\, R2)\, R3\, R4}{R3\,(R2 + R4) + R1\,(R2 + R3 + R4 + gm\, R3\, R4)}$$

In[6]:=
```
myGain //. {R1 -> 10^3, R2 -> 10^3, R3 -> 10^3, R4 -> 10^3, gm -> 0.1}
```

Out[6]=
```
-0.942857
```

In[7]:=
```
Row[{
  Button["Open Exercise_1.1-4.asc", SystemOpen[
    "/Users/isaacpabraham/Documents/Analog-Circuit/Teaching/2026_WSU_EE_758_AU/FA_
      2026/LTspice/Problem_1p1_4.asc"]]
 }]
```

Out[7]=
```
Open Exercise _1.1-4.asc
```

Let us generate a set of gains for different gm and R3. Program the value(s) into LTspice and check your result.

In[8]:=
```
myGain //. {R1 -> 10^3, R2 -> 10^3, R3 -> #1, R4 -> 10^3, gm -> #2} &[10 000, 0.]
```

Out[8]=
```
0.3125
```

2.2 Exercise 1.1-7 (pp 17)

2.2.1 Statement

1.1-7. Find v_2/v_1 and v_1/i_1 of Fig. P1.1-7.

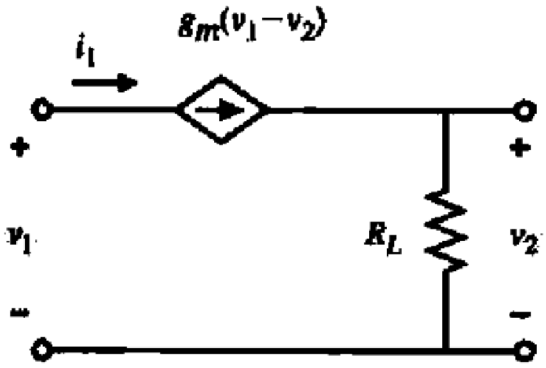


Figure P1.1-7

2.2.2 Solution - Part 1

The **first part** asks for v_2 / v_1 which is the forward gain signal G. How do I know that?

Ans ▼. Check the table that explains notation.

TABLE 1.2-1 Definition of the Symbols for Various Signals

Signal Definition	Quantity	Subscript	Example
Total instantaneous value of the signal	Lowercase	Uppercase	q_A
dc Value of the signal	Uppercase	Uppercase	Q_A
ac Value of the signal	Lowercase	Lowercase	q_a
Complex variable, phasor, or rms value of the signal	Uppercase	Lowercase	Q_a

The unsaid assumption is that the measuring devices do not load the circuit. All current (in the circuit) flows through R_L.

In[9]:=
```
Solve[v2 == gm (v1 - v2) * RL, v2][[1]]
```

Out[9]=
$$\left\{\text{v2} \to \frac{\text{gm RL v1}}{1 + \text{gm RL}}\right\}$$

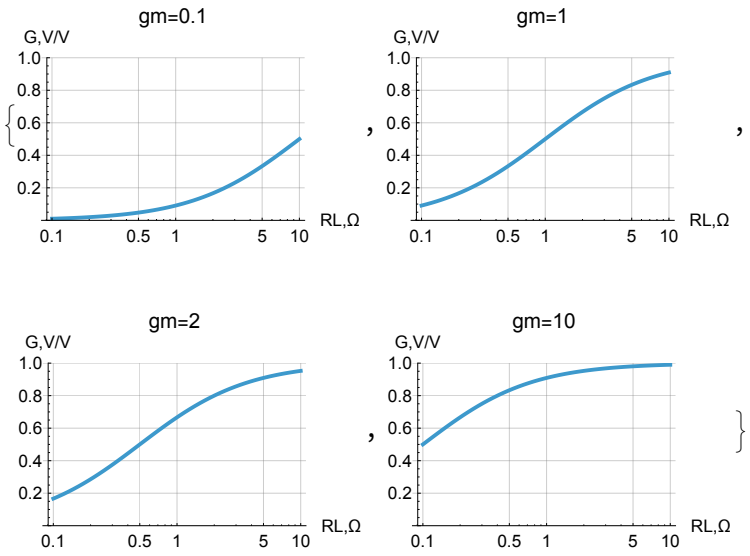
In[10]:=
```
G = (v2 //. Solve[v2 == gm (v1 - v2) * RL, v2][[1]]) / v1
```

Out[10]=

$$\frac{gm\ RL}{1 + gm\ RL}$$

In[11]:=
```
Column[{LogLinearPlot[G //. gm → #, {RL, 0.1, 10},
      AxesLabel → {"RL,Ω", "G,V/V"},
      PlotLabel → StringJoin["gm=", ToString[#]], PlotRange → {All, {0, 1}},
      GridLines → Automatic, ImageSize → Small] & /@ {0.1, 1, 2, 10}}]
```

Out[11]=



Out[12]=

LTspice

2.2.3 Solution - Part 2

The **second part** asks for the input resistance because the question wants us to find a ratio of voltage to current.

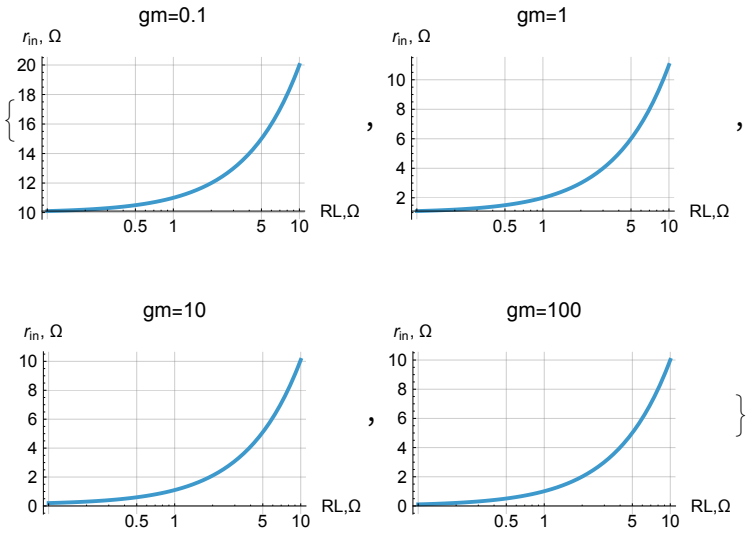
$$\frac{v_1}{i_1} = \frac{v_1}{gm * (v_1 - v_2)}$$

Divide the RHS by v_1. Recognize that $v_2 / v_1 \rightarrow G$ from part 1.

$$\frac{v_1}{i_1} = \frac{1}{gm * (1 - G)}$$

In[13]:=
```
Column[{LogLinearPlot[1/(gm * (1 - G)) //. {G → gm RL/(1 + gm RL)}, gm → #], {RL, 0.1, 10},
    AxesLabel → {"RL,Ω", "r_in, Ω"},
    PlotLabel → StringJoin["gm=", ToString[#]], PlotRange → Automatic,
    GridLines → Automatic, ImageSize → Small] & /@ {0.1, 1, 10, 100}}]
```

Out[13]=



Out[14]=

LTspice

We have an equation and a plot. Does this agree with intuition? Explain.

In[15]:=
```
rin = FullSimplify[1/(gm (1 - G)) //. {G → gm RL/(1 + gm RL)}]
```

Out[15]=

$$\frac{1}{gm} + RL$$

For (relatively) *high* values of gm, the input resistance
$r_{in} \rightarrow \boldsymbol{R_L}$ + small potential difference / some current) $\rightarrow \boldsymbol{R_L}$. We can expect this because the output and input voltages will be closely matched (recall the properties of a current source/sink).

For (relatively) *low* values of gm, the input resistance
$r_{in} \rightarrow (R_L$ + **large** potential difference / some current) $\rightarrow$ **large**.

3. References

1. Allen, P. and Holberg, D., CMOS Analog Circuit Design, Ed. 3.

4. Cite as

Out[ ]=

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