Math 275

Gibbs Measures and their Geometric Applications

Gibbs measures first arose in the formalism of statistical mechanics. A Gibbs measure is determined, via the Boltzmann hypothesis, by an energy function on the configuration space. The central object of the formalism is the partition function (or "pressure"). This is a function on the space of all possible energy functions (or, in particular models, on some finite dimensional subspace) whose derivatives encode both the Gibbs measure itself and its correlation properties.

Gibbs measures were applied in a purely mathematical setting in the work of Sinai on the ergodic theory of hyperbolic dynamical systems. In the dynamics setting, the energy function is replaced by a cocycle over the dynamics, and the partition function is traditionally called the "pressure". The theory of Sinai was further developed by Bowen, Ruelle, Walters, and others as a major tool for understanding the distribution of orbits for hyperbolic dynamical systems. In recent years, Gibbs measures have found significant application in the context of geometric and deformation properties of dynamical systems, and also in the more general setting of a geometric object with an associated equivalence relation, e.g. a smooth foliation or the limit set of a Fuchsian group.

The first part of this course will be an introduction to the basic formalism of Gibbs measures, both in the original statistical mechanics setting, and in the mathematical setting. We will then consider geometric applications of Gibbs measures to e.g. hyperbolic mappings, rational maps and quadratic polynomials, renormalization in dynamics, Kleinian and Fuchsian groups, Teichmüller spaces, Anosov flows on 3-manifolds, and subshifts of finite type.